Implementation of the dynamic Allan variance for the Galileo System Test Bed V2

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***Abstract*— The Dynamic Allan variance (DAVAR) method has been recently proposed as an extension of the classical Allan variance, with the aim of defining the instantaneous stability of an atomic clock. It is in fact known that atomic clocks may undergo sudden failures, are influenced by environmental factors, and they eventually age and stop working. Hence the stability of an atomic clock may vary with time, and in some demanding applications, such as Navigation Systems, a quick identification of clock failures is of the utmost importance. Working on the first preliminary data coming from the Galileo System Test Bed (GSTB) V2, some criticalities were encountered in the DAVAR evaluation and its graphical representation, when dealing with long periods of missing data. A new estimation method has been developed, allowing to evaluate the DAVAR in case of missing data, not equally spaced data and with large periods of missing observation. The new estimation method is presented here. The experimental results obtained by applying the improved DAVAR on the Galileo experimental satellite are also shown. Also experimental data obtained by using the Two-Way Satellite Time and Frequency Transfer (TWSTFT) method are analyzed and discussed.**

I. Introduction

Atomic clocks and oscillators can vary their behavior with time due to sudden failures, aging, change in physical quantities that can influence the clock stability, such as temperature. It is therefore important to introduce a representation that can take into account the time-varying nature of the stability of an oscillator. A new quantity has been recently proposed, the dynamic Allan variance, or DAVAR, that is a representation of the instantaneous stability of an atomic clock [2], [3].

The DAVAR is currently used in the frame of the Galileo System Test Bed V2 (GSTB V2), for the characterization of the clock on board of the first experimental Galileo satellite, GIOVE A. The GSTB V2 is the second experimental phase of the Galileo project supported by the European Space Agency (ESA), with the aim of mitigating the programme risks. Within the Galileo System Test Bed Version 2 project, two experimental satellites called GIOVE-A and GIOVE-B are being launched. They will mark the first step in the validation of the Galileo system to be completed with the deployment of the In-Orbit Validation satellites. The GIOVE-A satellite has been developed by Surrey Satellite Technology Ltd. (UK). GIOVE-A was launched from the Baikonur cosmodrome by a Soyuz rocket on December 28^{th} , 2005 and carries two Rubidium atomic clocks. GIOVE-B has been built by the Galileo Industries consortium, and its launch is scheduled by the end of 2007. It will fly a Passive Hydrogen Maser and a Rubidium clock.

Precise evaluation of the performance of the Rubidium Atomic Frequency Standard on-board GIOVE A has been the main objective driving this first period of the GSTB V2 experimentation [4]. In this context the stability estimation of the on board clocks presents some peculiar aspects which are not commonly encountered when the clock data are measured in a laboratory. The Galileo satellite has long periods of no visibility, and hence long periods of missing data. In addition, measurements may undergo several types of failures, so that the available measures are rich of missing data, even when the satellite is visible.

To cope with this experimental situation, the estimation of the DAVAR and its interpretation requests some effort. It is necessary to extend the definition of the dynamic Allan variance to the case of missing data, with large period of missing observation.

II. The DAVAR with missing data

The dynamic Allan variance has been originally proposed for complete data, that is without missing data. Unfortunately, in the experimental case, measures are very often rich of missing data. In the next sections we first review the definition of the DAVAR for complete data, and then we extend it to the case of missing data.

A. The classical DAVAR

The dynamic Allan variance is an extension of the Allan variance [1] that allows to represent the instantaneous stability of a clock. While the Allan variance $\sigma_y^2(\tau)$ is a two-dimensional representation that gives the stability of the clock on a given observation interval τ and for all times, the dynamic Allan variance $\sigma_y^2(t,\tau)$ is a three-dimensional surface that defines the stability of the clock at a given time t and for a given observation interval τ. The DAVAR is simply obtained by sliding the Allan variance on the data. The collection of all the Allan variances obtained gives the dynamic Allan variance $\sigma_y^2(t,\tau)$.

For discrete time phase data $x[n]$ the DAVAR can be written

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in the following form [3]

$$\sigma_{\boldsymbol{y}}^2[n,k] = \frac{1}{2k^2\tau_0^2}\frac{1}{N-2k}\times \sum_{m=n-N/2+k}^{n+N/2-k-1} E\left[(\boldsymbol{x}[m+k]-2\boldsymbol{x}[m]+\boldsymbol{x}[m-k])^2\right] \quad (1)$$

where $n = t/\tau_0$ is the discrete time, and $k = \tau/\tau_0$ is the observation interval in discrete time. It is

$$k = 0, 1, \ldots, \frac{N}{2} - 1 \quad (2)$$

The quantity N represents the window length. A short window allows to track fast variations of the data, but results in a lower quality of the estimate. On the contrary, a long window guarantees a better estimate, but does not allow a suitable localization of the events. The DAVAR is defined through an ensemble average, which means that $\sigma_{\boldsymbol{y}}^2[n,k]$ is a deterministic quantity. The square root of the dynamic Allan variance is the dynamic Allan deviation $\sigma_{\boldsymbol{y}}[n,k]$, or DADEV.

The DAVAR can be estimated from experimental data by using the following formula

$$\hat{\sigma}_{\boldsymbol{y}}^2[n,k] = \frac{1}{2k^2\tau_0^2}\frac{1}{N-2k}\times \sum_{m=n-N/2+k}^{n+N/2-k-1} (\boldsymbol{x}[m+k]-2\boldsymbol{x}[m]+\boldsymbol{x}[m-k])^2 \quad (3)$$

which is identical to the definition without the expectation value.

B. Extension to missing data

In case of missing data two approaches are possible:

1) The missing data can be reconstructed by using a proper interpolation technique. Then the classical DAVAR can be used on the complete data obtained. The disadvantage of this approach is that the DAVAR is biased, because the Allan variance at each time is biased [10].
2) The analysis can be based only on the available data, that is on the data with gaps. In this case an improved method must be developed in order to deal with the missing data.

We decided to follow the second approach, because the estimation is free of any possible bias due to the data manipulation. In implementing the dynamic Allan variance for missing data, we start by noticing that the DAVAR is, at any given time n, an Allan variance. The Allan variance is obtained by computing triplets of phase data, and by averaging their value squared. This can be seen in Eq. (3), where the generic triplet is given by $\boldsymbol{x}[m+k]-2\boldsymbol{x}[m]+\boldsymbol{x}[m-k]$. We say that a triplet is *complete* when all the three values $\boldsymbol{x}[m+k]$, $\boldsymbol{x}[m]$ and $\boldsymbol{x}[m-k]$ are available, and *incomplete* otherwise. In our method we discard all the incomplete triplets, and therefore we average the complete squared triplets only, to form the dynamic Allan variance at time n.

We point out that, for a given observation interval k, the number of complete triplets is, in general, a function of the time n. The reason is that the DAVAR is obtained by sliding the Allan variance on a window of length N centered about the discrete time n. In general, on two different windows, the number of complete triplets for a given k is different, which means that the number of complete triplets becomes a function of n. Therefore the quality of the estimation of the DAVAR changes with time, and will be better when the number of complete triplets is larger. We are currently working on a graphical representation that can put together the DAVAR along with a confidence surface that can take into account such variability of the estimation quality.

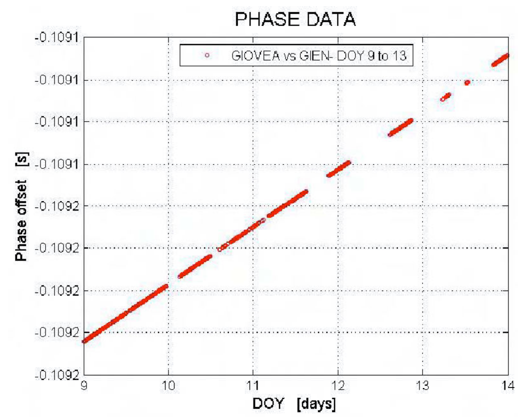

Fig. 1. Preliminary GIOVE-A apparent clock data.

When computing the DAVAR with our method it can happen that for a given time n all the triplets for all observation intervals are incomplete. In such case the DAVAR at time n will be completely missing. This can happen for several time instants, generating a "hole" in the DAVAR surface. To cope with this problem, the three-dimensional representation of the DAVAR has been optimized to properly show "canyons" where the missing regions are.

A free implementation in MATLAB® [1] language of the dynamic Allan variance for missing data can be found at the address `www.ien.it/tf/ts/clock_behavior.shtml` .

III. Analysis of the data from the first experimental Galileo satellite GIOVE-A

The first experimental Galileo satellite, GIOVE-A, flies two Rubidium atomic clocks. Some of the first preliminary GIOVE-A apparent clock data are shown in Fig. 1. As can be seen, there are several blocks of missing data due to the periods of no visibility of the satellite. Since we are interested in characterizing the time-varying stability of the Rubidium clock, we have to compute the dynamic Allan variance in presence of missing data. We therefore apply the method described in Sect. II to the signal of Fig. 1. The corresponding dynamic Allan variance is shown in Fig. 2, where it can be noticed that canyons have been introduced when the DAVAR is missing because of large gaps in the experimental data.

[1]MATLAB is a registered trademark of The Mathworks Inc.

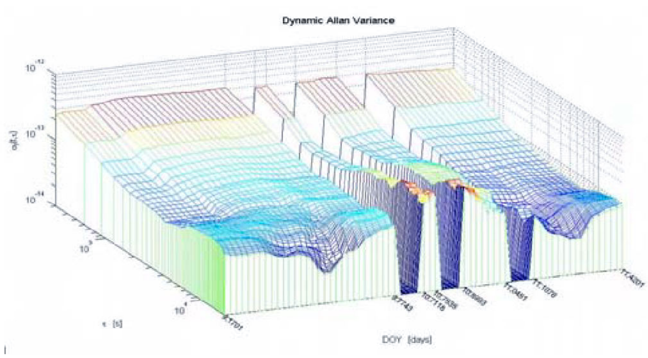

Fig. 2. Dynamic Allan deviation of the preliminary GIOVE-A apparent clock data shown in Fig. 1.

IV. Analysis of the TWSTFT data

The Two-Way Satellite Time and Frequency Transfer, or TWSTFT, is a method that allows to compare two remote clocks or time scales by means of a geostationary satellite. Using this method it was possible to compare the US Naval Observatory time scale, UTC(USNO), versus the Italian time scale, UTC(IT), during the period starting from January 1^{st}, 2007 and ending on May 15^{th}, 2007. In Fig. 3 we show the corresponding phase signal, namely UTC(USNO) vs. UTC(IT). We notice that there are missing data. Moreover, we seem to recognize a certain periodicity in the data.

To understand the time-varying stability of UTC(USNO) vs. UTC(IT), we have computed its dynamic Allan deviation, shown in Fig. 4. Two things can be pointed out from this picture:

1) A few bumps arise from the DADEV surface. The most evident one happens in the second block of DADEV data, roughly in the time interval ranging from MJD 54125 to MJD 54147 (MJD stands for Modified Julian Date).
2) An oscillation in time is visible for long observation intervals.

These two facts are connected to the periodicity of the phase data, probably due to the sensitivity to the temperature.of the equipment of the Two-Way stations involved in the remote clock comparison.

To understand how the DAVAR represents periodicities in the data, we have numerically generated a signal made by the sum of a white phase noise and two short duration sinusoids. The signal is represented in Fig. 5. The dynamic Allan variance depends on the choice of the window length N [3], especially in case of data with periodicities.

We have first computed the dynamic Allan deviation with a window length equal to the period of the sinusoidal oscillations, and we report it in Fig. 6. We see that the dynamic Allan deviation surface is stationary outside the oscillation intervals, and its slope for a given time corresponds to a white phase noise as expected. During the intervals in which the sinusoids are present, we observe an oscillation in time of the DADEV for long observation intervals.

We then computed the dynamic Allan deviation for a window with a length ten times the period of the short duration sinusoids. Fig. 7 shows the computed DADEV. In this case the oscillation in time for long observation intervals have disappeared, because they have been averaged out by the long window. We instead see a series of bumps that represent the effect of the periodicities.

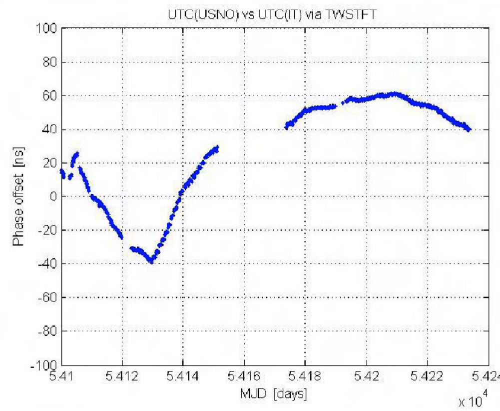

Fig. 3. Remote clocks comparison with TWSTFT technique: UTC(USNO) vs UTC(IT) from 01/01/07 to 15/05/07.

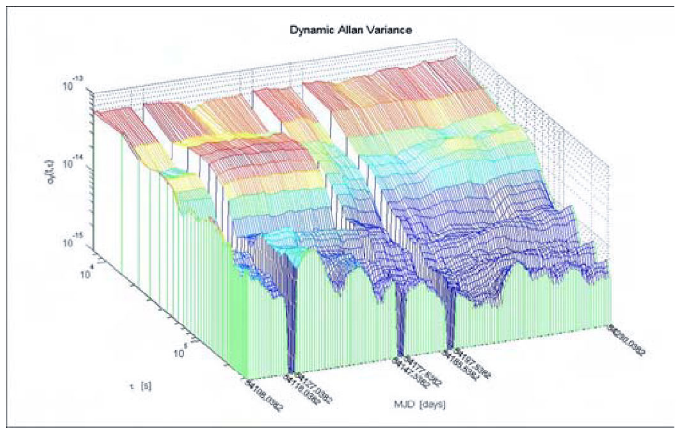

Fig. 4. Dynamic Allan deviation of the phase signal shown in Fig. 3.

If the window length is within the two extreme conditions described above, then we expect the corresponding DAVAR to show a combination of the two effects, namely the oscillation in time for long observation intervals and the bumps. This is precisely what happens in the TWSTFT case shown in Fig. 4. As a consequence of the performed analysis, we stress the fact that attention has to be paid in the choice of the window length.

V. Conclusion

The dynamic Allan variance is a representation of the time-varying stability of an atomic clock. It is currently used in the frame of the GSTB V2 for the characterization of the clock on board the first experimental Galileo satellite GIOVE-A. Since GIOVE-A has long periods of no visibility, the measures are rich of missing data. To cope with this experimental situation,

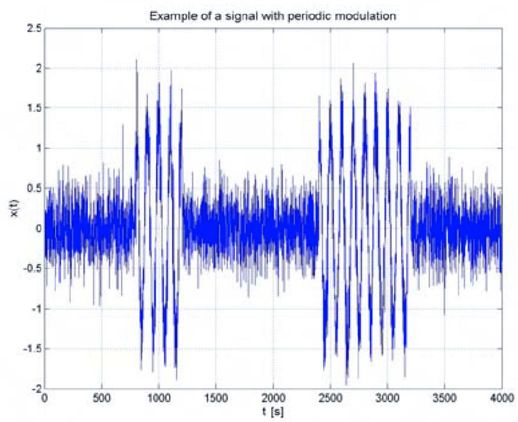

Fig. 5. Numeric signal made by the sum of a white phase noise and two short duration sinusoids.

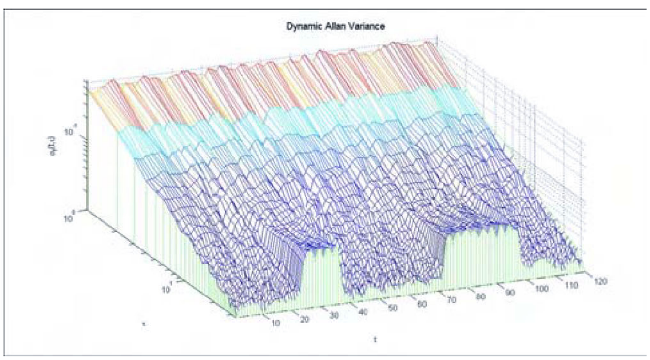

Fig. 6. Dynamic Allan deviation of the signal shown in Fig. 5. The window length of the DADEV equals the period of the sinusoids.

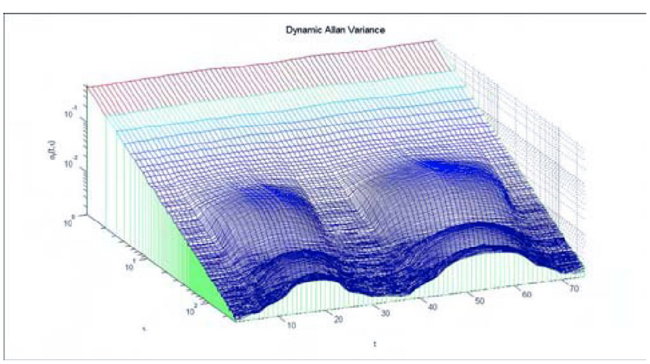

Fig. 7. Dynamic Allan deviation of the signal shown in Fig. 5. The window length of the DADEV is ten times the period of the sinusoids.

the dynamic Allan variance has been extended, allowing the estimation of the instantaneous stability in presence of missing data. We have described the implementation of such improvement, and we have applied the method to the first GIOVE-A data. We have also applied it to experimental data obtained by using the Two-Way Satellite Time and Frequency Transfer technique. Both applications prove that the dynamic Allan variance can describe the time-varying stability of the clock data even in presence of large periods of missing data.

The dynamic Allan variance has been implemented in the STABLE32 software (`www.wriley.com`) and in the CANVAS software, developed by the US NAVY (`https://goby.nrl.navy.mil/canvas/`).

VI. Acknowledgement

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